Steady Temperature Distributions of Both Fluids along Parallel Flow and Counter Flow Double Pipe Heat Exchangers

Abdul-rahman Ahmed Abu-shanab*
Mechanical Engineering Department, Higher Polytechnic Institute, Zliten, Libya
*Corresponding Author: abdulrahman_aboshanab@yahoo.com

Abstract

Mathematical relationships describe the dimensionless temperature profiles for both fluid streams along parallel and counter flow heat exchangers for steady state case will be developed for the two case. The first case when the heat capacity rate of a cold fluid is smaller than for the heat capacity rate of a hot fluid, the second case when the heat capacity rate of a cold fluid is greater than for the heat capacity rate of a hot fluid. The fluid temperatures along parallel and counter flow heat exchangers are found to be dependent on the magnitude of fluid inlet temperatures, the heat exchanger length, the heat capacity rate, and the number of transfer units, for both parallel flow and counter flow heat exchangers. An experimental work was performed on a double pipe heat exchanger to validate an analytical solution results. The fluid temperatures along parallel and counter flow heat exchangers are found to be increase by increasing of the hot fluid mass flow rate or by decreasing of the cold fluid mass flow rate. Results show a good agreement between analytical solution and experimental work, for the steady state behavior of parallel and counter flow heat exchangers.

Keywords: Heat exchangers; steady state; temperature distributions

1. Introduction

Steady–state solutions of heat exchangers are commonly based on the overall heat transfer coefficient [1, 3]. The LMTD method consists of calculating the logarithmic mean temperature difference. This method is a simple method to use when the inlet temperatures are known and the outlet temperatures are specified. If the outlet temperatures are unknown, then the effectiveness–NTU (number of transfer units) method should be used. This method is based on calculation of the maximum possible heat transfer rate and the number of transfer units. These methods give an overall approach for heat exchanger analysis. In some applications, the spatial temperature profile is required, particularly when local temperature calculation is necessary for safety analysis. In this case, the classical techniques cannot give satisfactory analysis for this kind of assessment in thermal equipment. Indeed, a partial approach is needed [4]. Abdelghani–Idrissi and Bagui [4] obtained governing mathematical relationships to describe the dimensionless temperature profiles of hot and cold fluids depending on the convective heat transfer coefficients of hot and cold fluids in the case of a counter-flow double pipe heat exchanger. Bhanuchandrarao et al. [5] use ANSYS FLUENT12.1 software and hand calculations to analyze the temperature drops as a function of both inlet velocity and inlet temperature and how each varies with the other in both parallel and counter flow double pipe heat exchangers. The present study was undertaken to develop governing mathematical relationships, which describe the dimensionless temperature profiles for both fluid streams along an insulated parallel and counter flow heat exchanger for steady state case for the two case. The first case when the heat capacity rate of a cold fluid is smaller than for the heat capacity rate of a hot fluid, the second case when the heat capacity rate of a cold fluid is greater than for the heat capacity rate of a hot fluid. They are depending on the magnitude of fluid inlet temperatures, the heat exchanger length L, the number of heat transfer unit, and heat capac-

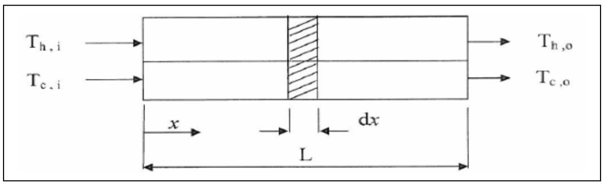

Figure 2.1: An installed parallel-flow heat exchanger

ity rate ratio C_r. The assumptions to be considered are as follows:

1. The heat exchanger is insulated from its surroundings, in which case the only heat exchange is between the hot and cold fluids
2. Steady state condition.
3. The fluid streams temperature varies only in the longitudinal direction.
4. The work done, potential and kinetic energy changes are negligible.
5. The specific heats as well as other fluid properties are constant.
6. The overall heat transfer coefficient is constant.
7. Longitudinal heat conduction in the walls is negligible.

2. Steady State Formulation

2.1. Governing Equations for the First Case

In this section the governing equations, which describe the dimensionless temperature distributions for both fluid streams along an insulated parallel and counter flow heat exchangers for steady state case, are derived when the heat capacity rate of a cold fluid is smaller than for the heat capacity rate of a hot fluid.

2.1.1. The Parallel-Flow Heat Exchanger

Consider a parallel-flow heat exchanger, which insulated from its surroundings, as shown in Figure 2.1. Where (T) temperature, (h) hot fluid, (c) cold fluid, (o) outlet, (i) inlet, and (x) longitudinal coordinate. Based on the assumptions mentioned above and, an energy balance on the differential element of the heat exchanger can be made to obtain the following equations for the hot and cold fluids.

$$\left[\dot{m}\, c_p T - \dot{m}\, c_p \left(T + \frac{dT}{dx} dx\right)\right]_h + \frac{UA}{L}\left(T_c - T_h\right) dx = 0 \tag{2.1}$$

$$\left[\dot{m}\, c_p T - \dot{m}\, c_p \left(T + \frac{dT}{dx} dx\right)\right]_c + \frac{UA}{L}\left(T_h - T_c\right) dx = 0 \tag{2.2}$$

Where $\dot{m}$ mass flow rate ($\dot{m} = \rho Q$), (ρ = density, Q = volumetric flow rate)
cp = Specific heat at constant pressure
U = Overall heat transfer coefficient
A = Heat transfer area
Simplifying Equations, (2.1) and (2.2) reduce to:

$$L\left(\frac{dT_h}{dx}\right) + C_r NTU \left(T_h - T_c\right) = 0 \tag{2.3}$$

$$L\left(\frac{dT_c}{dx}\right) + NTU \left(T_h - T_c\right) = 0 \tag{2.4}$$

Where:

$$NTU = \frac{UA}{C_{min}} = \frac{UA}{C_c} \quad \text{and} \quad C_r = \frac{C_c}{C_h}$$

C = Heat capacity rate ($C = \dot{m}cp$)
Introducing the dimensionless parameters:

$$\theta = \frac{T - T_{c,i}}{T_{h,i} - T_{c,i}} \quad \text{and} \quad X = \frac{x}{L}$$

Equations (2.3) and (2.4) can be written in dimensionless form as:

$$\frac{d\theta_h}{dx} + C_r NTU \left(\theta_h - \theta_c\right) = 0 \tag{2.5}$$

$$\frac{d\theta_c}{dx} + NTU \left(\theta_h - \theta_c\right) = 0 \tag{2.6}$$

The heat exchanger is subject to the following boundary conditions: $\theta_h = 1$ at $X = 0$ and $\theta_c = 0$ at $X = 0$
After solving the equations (2.5) and (2.6), expressions for θ_h and θ_c are:

$$\theta_h = \frac{1}{1 + C_r}\left[1 + C_r\, exp(-\alpha X)\right] \tag{2.7}$$

$$\theta_c = \frac{1}{1 + C_r}\left[1 - C_r\, exp(-\alpha X)\right] \tag{2.8}$$

Where:

$$\alpha = NTU\left(1 + C_r\right) \tag{2.9}$$

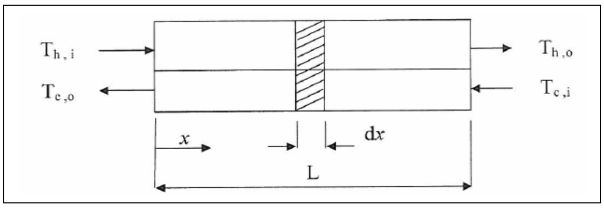

Figure 2.2: An installed counter-flow heat exchanger

2.1.2. The Counter Flow Heat Exchanger

Consider a counter-flow heat exchanger, which insulated from its surroundings, as shown in figure (2.2). Using the same assumptions and procedures as in the parallel-flow heat exchanger, an energy balance on the differential element of the heat exchanger gives the following equations for the hot and cold fluids:

$$\left[\dot{m}\, c_p T - \dot{m}\, c_p \left(T + \frac{dT}{dx} dx\right)\right]_h + \frac{UA}{L}\left(T_c - T_h\right) dx = 0 \tag{2.10}$$

$$-\left[\dot{m}\, c_p T - \dot{m}\, c_p \left(T + \frac{dT}{dx} dx\right)\right]_c + \frac{UA}{L}\left(T_h - T_c\right) dx = 0 \tag{2.11}$$

Simplifying equations, (2.10) and (2.11) reduce to:

$$L\left(\frac{dT_h}{dx}\right) + C_r NTU\left(T_h - T_c\right) = 0 \tag{2.12}$$

$$L\left(\frac{dT_c}{dx}\right) + NTU\left(T_h - T_c\right) = 0 \tag{2.13}$$

Equations (2.12) and (2.13) can be written in dimensionless form as:

$$\frac{d\theta_h}{dX} + C_r NTU\left(\theta_h - \theta_c\right) = 0 \tag{2.14}$$

$$\frac{d\theta_c}{dX} + NTU\left(\theta_h - \theta_c\right) = 0 \tag{2.15}$$

The heat exchanger is subject to the following boundary conditions: $\theta_h = 1$ at $X = 0$ and $\theta_c = 0$ at $X = 1$

After solving the equations (2.14) and (2.15), expressions for θ_h and θ_c are:

$$\theta_h = \frac{exp(\gamma) - C_r\, exp(\gamma\, X)}{exp(\gamma) - C_r} \tag{2.16}$$

$$\theta_c = \frac{exp(\gamma) - exp(\gamma\, X)}{exp(\gamma) - C_r} \tag{2.17}$$

Where:

$$\alpha = NTU\left(1 + C_r\right) \tag{2.18}$$

2.2. Governing Equations for the Second Case

In this section the governing equations, which describe the dimensionless temperature distributions for both fluid streams along an insulated parallel and counter flow heat exchangers for steady state case, are derived when the heat capacity rate of a cold fluid is greater than for the heat capacity rate of a hot fluid.

2.2.1. The Parallel-Flow Heat Exchanger

Consider the same parallel-flow heat exchanger which shown in Figure (2.1). Using the same assumptions, procedures, and boundary conditions as in the first case, an energy balance on the differential element of the heat exchanger gives the same equations (2.1) and (2.2) for the hot and cold fluids.

On simplifying equations, (2.1) and (2.2) reduce to:

$$L\left(\frac{dT_h}{dx}\right) + NTU\left(T_h - T_c\right) = 0 \tag{2.19}$$

$$L\left(\frac{dT_c}{dx}\right) - C_r NTU\left(T_h - T_c\right) = 0 \tag{2.20}$$

Where:

$$NTU = \frac{UA}{C_{min}} = \frac{UA}{C_h} \quad \text{and} \quad C_r = \frac{C_h}{C_c}$$

Equations (2.19) and (2.20) can be written in dimensionless form as:

$$\frac{d\theta_h}{dX} + NTU\left(\theta_h - \theta_c\right) = 0 \tag{2.21}$$

$$\frac{d\theta_c}{dX} - C_r NTU\left(\theta_h - \theta_c\right) = 0 \tag{2.22}$$

After solving the equations (2.21) and (2.22), expressions for θ_h and θ_c are:

$$\theta_h = \frac{1}{1 + C_r}\left[C_r + exp(-\alpha X)\right] \tag{2.23}$$

$$\theta_c = \frac{C_r}{1 + C_r}\left[1 - exp(-\alpha X)\right] \tag{2.24}$$

2.2.2. The Counter Flow Heat Exchanger

Consider the same counter-flow heat exchanger which shown in Figure (2.2). Using the same assumptions, procedures, and boundary conditions as in the first case, an energy balance on the differential element of the heat exchanger gives the same equations (2.10) and (2.11) for the hot and cold fluids.
Simplifying equations, (2.10) and (2.11) reduce to:

$$L\left(\frac{dT_h}{dx}\right) + NTU\left(T_h - T_c\right) = 0 \tag{2.25}$$

$$L\left(\frac{dT_c}{dx}\right) - C_r NTU\left(T_h - T_c\right) = 0 \tag{2.26}$$

Equations (2.25) and (2.26) can be written in dimensionless form as:

$$\left(\frac{d\theta_h}{dx}\right) + NTU\left(\theta_h - \theta_c\right) = 0 \tag{2.27}$$

$$\left(\frac{d\theta_c}{dx}\right) - C_r NTU\left(\theta_h - \theta_c\right) = 0 \tag{2.28}$$

After solving the equations (2.27) and (2.28), expressions for θ_h and θ_c are:

$$\theta_h = \frac{C_r\, exp(-\gamma) - exp(-\gamma\, X)}{C_r\, exp(-\gamma) - 1} \tag{2.29}$$

$$\theta_c = \frac{C_r\left[exp(-\gamma) - exp(-\gamma\, X)\right]}{C_r\, exp(-\gamma) - 1} \tag{2.30}$$

The Nusselt numbers are calculated from correlations presented in [6]. For laminar flow in circular annular duct:

$$Nu = Nu_\infty + \left[1 + 0.14\left(\frac{d_o}{D_i}\right)^{-0.5}\right] \times \frac{0.19\left(\frac{Pe_b D_h}{L}\right)^{0.8}}{1 + 0.117\left(\frac{Pe_b D_h}{L}\right)^{0.467}} \tag{2.31}$$

Where:
Pe_b = Peclet number at bulk fluid conditions
$Pe = Re\,Pr$
Re = Reynolds number
Pr = Prandtl number
d_o = outer diameter of inner tube
Di = inner diameter of a circular annulus
Dh = hydraulic diameter of a tube
Nu_∞ = Nusselt number for fully developed flow

$$Nu_\infty = 3.66 + 1.2\left(\frac{d_o}{D_i}\right)^{-0.5} \tag{2.32}$$

Where the outer wall of the annulus is insulated. For turbulent flow:

$$Nu = \frac{(f/2)\, Re_b\, Pr_b}{1.07 + 12.7\,(f/2)^{0.5}\left(Pr_b^{2/3} - 1\right)} \tag{2.33}$$

Equation (2.32) predicts the results in the range $\left(10^4 < Re_b < 5106\right)$ and $(0.5 < Pr_b < 200)$. In the transition region where Reynolds numbers are between 2300 and 10^4 :

$$Nu = \frac{(f/2)\,(Re_b - 1000)\, Pr_b}{1 + 12.7\,(f/2)^{0.5}\left(Pr_b^{2/3} - 1\right)} \tag{2.34}$$

Where:

$$f = (1.58\, ln\,(Re_b) - 3.28)^{-2} \tag{2.35}$$

Where f = Flonenko's friction factor

3. Experimental Device

The experimental test device used to characterize the distribution of steady-state temperatures of hot and cold fluids is presented schematically in Figure (3.1). The temperatures of the hot water entering and leaving the heat exchanger are measured by two thermocouples of type T. Temperature probes, corresponding of thermocouples of type T, are placed along the outer tube of the insulated heat exchanger with a distance of 0.4 m from each other, with an accuracy of ± 0.3 °C. The volume flow rates of hot and cold water were measured by two flow meters 1 and 2. The range of measurement of each flow meter is from 0 to 300 Lt/hr, with an accuracy of ± 2 Lt/hr. The inner tube of the double pipe heat exchanger is made from copper and the outer one is made from galvanized cast iron. The physical and geometrical characteristics of the tubes are reported in Table 3.1. A series of valves 1, 2, 3 and 4 are used to select parallel or counter flow heat exchanger.

4. Results and Discussion

The mathematical relationships that derived are used to study a steady-state behavior of parallel flow and counter flow heat exchangers. They were applied on a water-water concentric tube heat exchanger that

Table 3.1: Obtained parameters of feedback controller

Tube	Thermal Conductivity (W/m °C)	Specific Heat (J/kg °C)	Density (kg/m^3)	Outer Diameter (m)	Inner Diameter (m)	Length (m)
Inner	348	394	8900	0.0142	0.0126	2.2
Outer	52	420	7272	0.034	0.0284	2.2

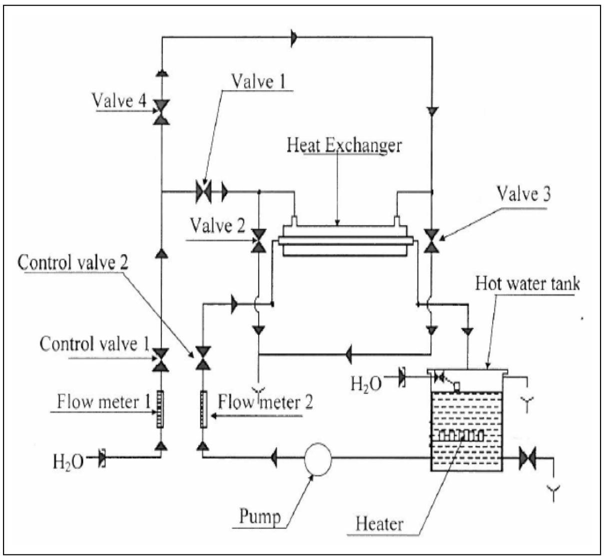

Figure 3.1: The experimental test device

described above. The inlet temperatures of hot and cold water are 42 °C and 23 °C respectively. The steady behavior of both fluids along the heat exchanger is shown in Figures (4.1) to (4.4) for a parallel flow arrangement. It can be seen in Figures (4.1) and (4.2) that with decreasing of a cold fluid volume flow rate from 200 to 100 Lt/hr, an increasing in both hot and cold water temperature distributions along the heat exchanger can be occurred. Figures (4.3) and (4.4) show that with increasing of a hot fluid volume flow rate from 150 to 300 Lt/hr, an increasing in both hot and cold water temperature distributions along the heat exchanger can be occurred.

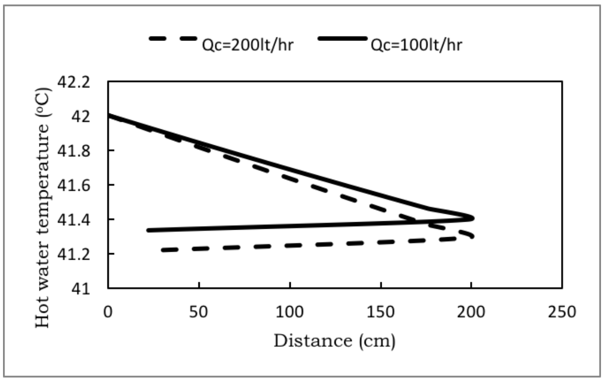

Figure 4.1: Temperature distribution of the hot fluid along the parallel-flow heat exchanger at $Q_h = 250$ Lt/hr and $Q_c =$ 100, and 200 Lt/hr

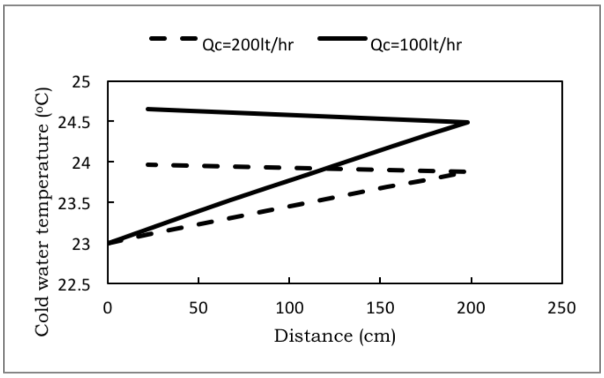

Figure 4.2: Temperature distribution of the cold fluid along the parallel-flow heat exchanger at $Q_h = 250$ Lt/hr and $Q_c =$ 100, and 200 Lt/hr

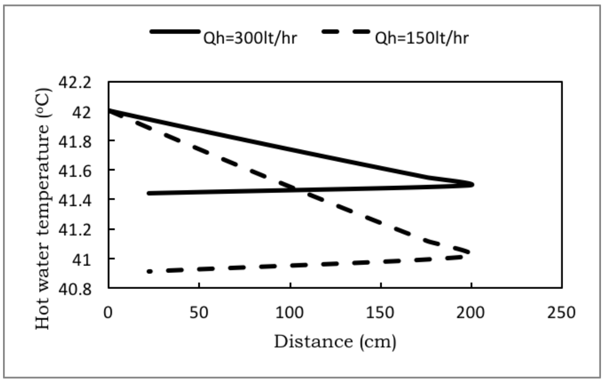

Figure 4.3: Temperature distribution of the hot fluid along the parallel-flow heat exchanger at $Q_c = 100$ Lt/hr and $Q_h = 150$, and 300 Lt/hr

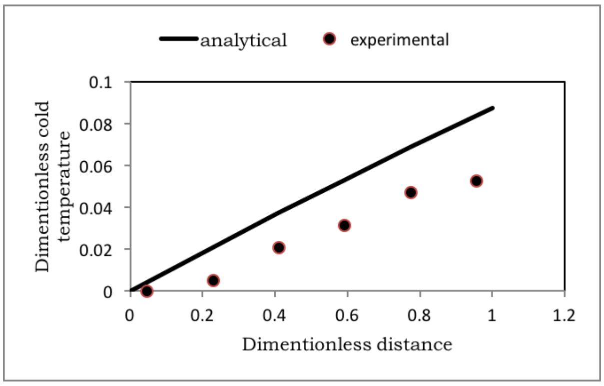

Figure 4.5: Experimental and analytical steady-state dimensionless temperature of cold fluid along parallel-flow heat exchanger at $Q_h = 300$ Lt/hr and $Q_c = 100$ Lt/hr

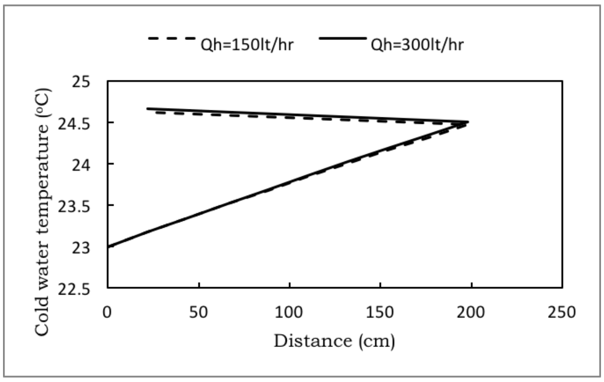

Figure 4.4: Temperature distribution of the cold fluid along the parallel-flow heat exchanger at $Q_c = 100$ Lt/hr and $Q_h = 150$, and 300 Lt/hr

Table 4.1 shows a comparison between the steady behavior of the parallel and counter flow heat exchangers. The cold fluid outlet temperature, the heat transfer rate, and the heat exchanger effectiveness for the counter flow heat exchanger are greater than those for the parallel flow heat exchanger, but a hot fluid outlet temperature of the parallel flow heat exchanger is greater than that for the counter flow heat exchanger. The comparison between analytical and experimental results of the dimensionless temperature distributions for the cold fluid along the parallel flow heat exchanger is shown in Figure (4.5) at $Q_h = 300$ L/hr and $Q_c = 100$ L/hr, and in Figure (4.6) along the counter flow heat exchanger at $Q_h = 150$ L/hr and $Q_c = 200$ L/hr. All figures show a good agreement between analytical and experimental results.

Equations (2.7), (2.8), (2.16) and (2.17) were applied to the heat exchangers described as sample problem 11.2 in [1]. The outlet temperature for cold and hot fluids are found 119.52 °C and 134.29 °C respectively in the case of the parallel flow heat exchanger, and are found 139.34 °C and 121.20 °C in the case of the counter flow heat exchanger, which are the same answers given in the sample problem 11.2. Equations (2.29) and (2.30) were applied to the heat exchanger described as example 11.1 in [3]. The outlet temperatures for cold and hot fluids are found 40.21 °C and 59.99 °C respectively, which are the same characteristics given in the sample example 11.1

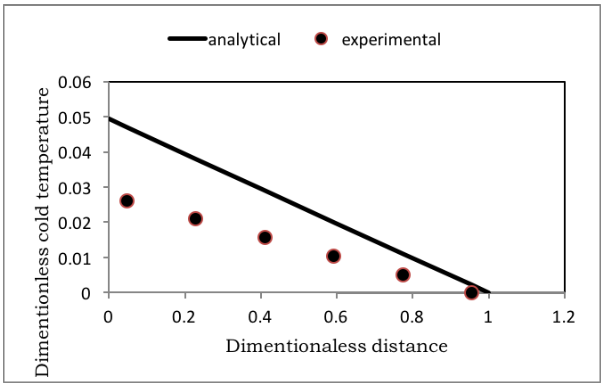

Figure 4.6: Experimental and analytical steady-state dimensionless temperature of cold fluid along counter-flow heat exchanger at $Q_h = 150$ Lt/hr and $Q_c = 200$ Lt/hr

5. Conclusion

This work presents an experimental work which was validated using a very well established mathematical expression for heat transfer in double pipe heat exchanger design. The dimensionless temperature

Table 4.1: The heat exchanger performance at $Q_h = 150$ Lt/hr and $Q_c = 250$ L/hr

Flow Type	Cold Fluid Inlet Temp (°C)	Cold Fluid Outlet Temp (°C)	Hot Fluid Inlet Temp (°C)	Hot Fluid Outlet Temp (°C)	Heat Transfer rate (W)	Heat Exchanger Effectiveness
Parallel	23	23.79097	42	40.67276	229.028	0.069855
Counter	23	23.79178	42	40.67139	229.263	0.069927

profiles for both fluid streams along parallel and counter flow heat exchangers are based on the value of fluid inlet temperatures, the heat capacity rate, the number of transfer units, and the heat exchanger length. The fluid temperatures along parallel and counter flow heat exchangers increase by increasing of the hot fluid mass flow rate or by decreasing of the cold fluid mass flow rate. The cold fluid outlet temperature, the heat transfer rate, and the heat exchanger effectiveness of the counter flow heat exchanger are greater than those for the parallel flow heat exchanger, but the hot fluid outlet temperature for the counter flow heat exchanger is smaller than that for the parallel flow heat exchanger at the same operation conditions.

References

[1] Karlecar, B. V.; Desmond, R. M. Heat Transfer. West publishing Co., Minnesota. 1982, 2nd ed., chapter 11, pp. 689-752.

[2] Holman, J. P. Heat Transfer. McGraw-Hill, New York. 2010, 10th ed., chapter 10, pp. 521-585.

[3] Bergman, T. L.; Lavine, A. S.; Incropera, F. P.; DeWitt, D. P. Fundamentals of Heat and Mass Transfer. Wiley, New York. 2011, 7th ed., chapter 11, pp. 705-765.

[4] Abdelghani Idrissi, M. A.; Bagui, F. Counter Current Double–Pipe Heat Exchanger Subjected to Flow–Rate Step Change, Part 1: New Steady–State Formulation. Heat Transfer Engineering, 2002, Vol. 23, pp. 4-11.

[5] Bhanuchandrarao, D.; Chakravarthy, M.; Krishna, Y.; Rao, V.; Krishna, T. CFD Analysis and Performance of Parallel and Counter Flow in Concentric Tube Heat Exchangers. International Journal of Engineering Research & Technology, 2013, Vol. 2, pp. 2782-2792.

[6] Kakac, S.; Liu, H. Heat Exchangers: Selection, Rating, and Thermal Design. CRC Press, Boca Raton, FL. 1998, chapter 3, pp. 73-113.